

- **1** Section 8.1: integration by parts
- **2** Section 8.2: trigonometric integrals

Integrating the product rule

$$\int f'(x)g(x) \, \mathrm{d}x = f(x)g(x) - \int f(x)g'(x) \, \mathrm{d}x$$

**Proof:** 

■ Recall the product rule for differentiation:

$$\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x).$$

Integrating both sides gives

$$\int \frac{d}{dx} f(x)g(x) \, \mathrm{d}x = \int f'(x)g(x) + f(x)g'(x) \, \mathrm{d}x$$
$$f(x)g(x) = \int f'(x)g(x) \, \mathrm{d}x + \int f(x)g'(x) \, \mathrm{d}x$$

Moving the second term to the left gives the boxed formula.

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Integrating the product rule

For definite integrals the rule reads as

$$\int_{a}^{b} f'(x)g(x) \, \mathrm{d}x = [f(x)g(x)]_{a}^{b} - \int_{a}^{b} f(x)g'(x) \, \mathrm{d}x$$
$$= f(b)g(b) - f(a)g(a) - \int_{a}^{b} f(x)g'(x) \, \mathrm{d}x.$$

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$$\int x e^x \, \mathrm{d}x = ??$$

Example 1: choose wisely...

$$\int x e^x \, \mathrm{d}x = ??$$

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Explicit renaming

$$\int f'(x)g(x) \, \mathrm{d}x = f(x)g(x) - \int f(x)g'(x) \, \mathrm{d}x$$

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Notate f(x) = u and g(x) = v, then the rule becomes

$$\int u'v \, \mathrm{d}x = uv - \int u \, v' \, \mathrm{d}x$$

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Example 1, alternative 2

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$$\int x e^x \, \mathrm{d}x = ??$$

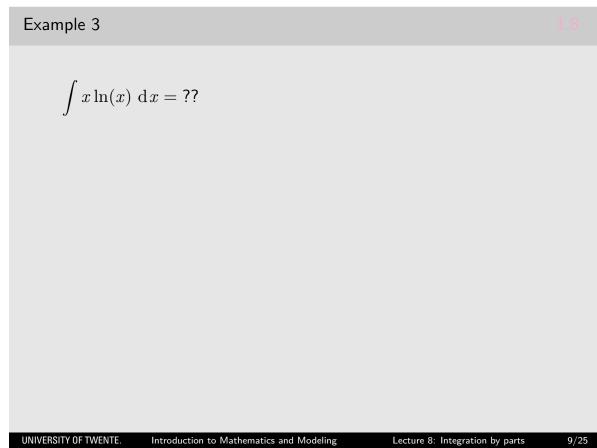
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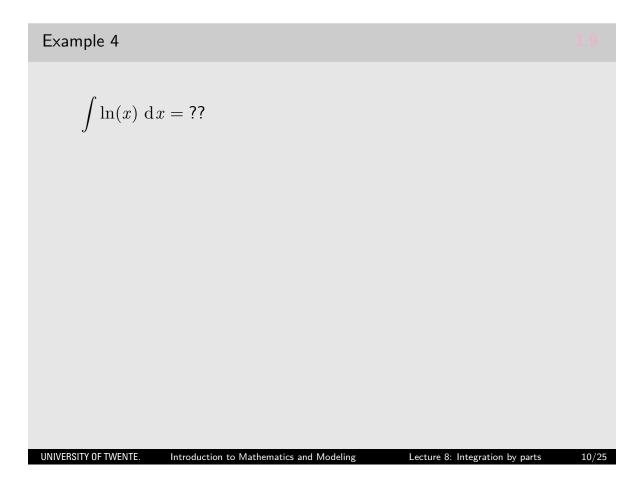
1.5

Example 2 1.7  

$$\int x^2 e^{-x} dx = ??$$
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Implicit integration by parts

• With the explicit renaming f(x) = u and g(x) = v:

$$\int u'v \, \mathrm{d}x = uv - \int u \, v' \, \mathrm{d}x$$

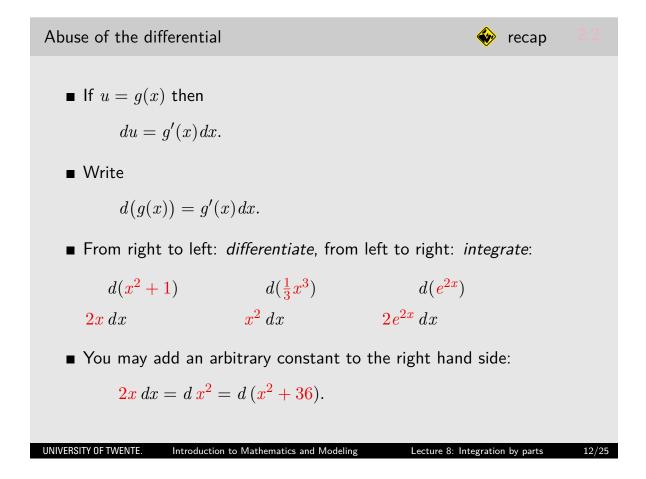
■ Note that du = u' dx and dv = v' dx. Therefore the rule can be memorized as follows:

$$\int v \, \mathrm{d} u = uv - \int u \, \mathrm{d} v$$

• You can even do this *without* renaming *f* and *g*:

$$\int g(x) \, \mathrm{d}f(x) = f(x)g(x) - \int f(x) \, \mathrm{d}g(x)$$

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Example 5 2.3  $\int (2x+1)e^x \, dx = ??$ 

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Example 6, first attempt

$$\int x^3 e^{x^2} \, \mathrm{d}x = ??$$

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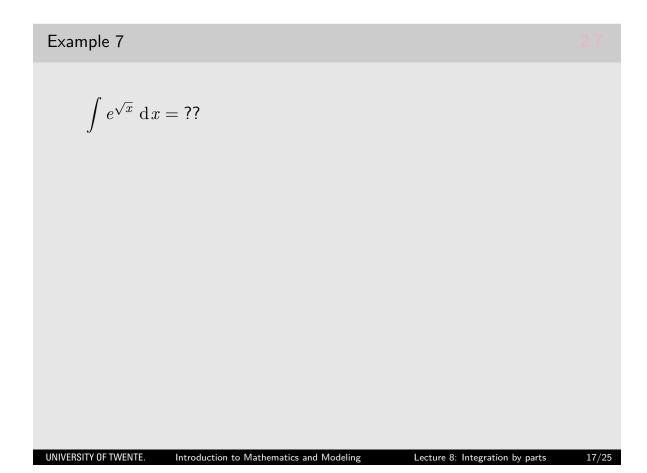
Example 6, alternative 1

$$\int x^3 e^{x^2} \, \mathrm{d}x = ??$$

Example 6, alternative 2

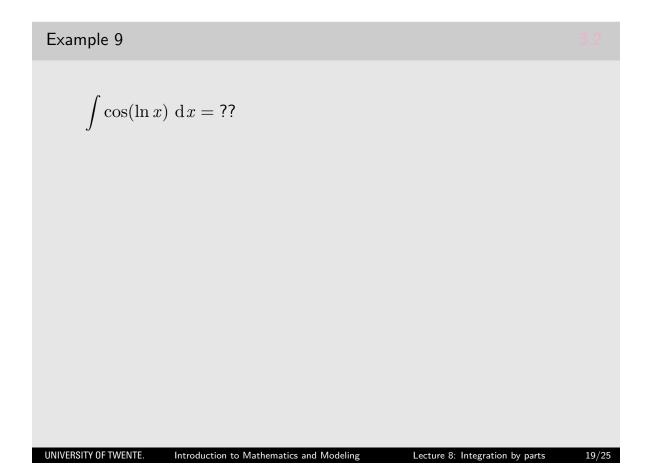
$$\int x^3 e^{x^2} \, \mathrm{d}x = ??$$

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2.6

Example 8 
$$I = \int e^x \cos(x) \, dx = ??$$



Let m and n be non-negative integers.

$$\int \sin^m x \cos^n x \, \mathrm{d}x = ??$$

The following formulas are useful:

$$- \quad \sin^2 x + \cos^2 x = 1$$

$$- \quad \sin x \cos x = \frac{1}{2} \sin(2x)$$

$$- \quad \sin^2 x = \frac{1}{2} (1 - \cos(2x))$$

$$-\cos^2 x = \frac{1}{2}(1 + \cos(2x))$$

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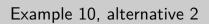
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Example 10, alternative 1

$$\int \sin^2 x \cos^2 x \, \mathrm{d}x = ??$$

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$$\int \sin^2 x \cos^2 x \, dx = ??$$

